Excitation of Kaluza-Klein modes of $U(1)$ field by parametric resonance

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In this paper, based on the theory of parametric resonance, we propose a cosmological criterion on ways of compactification: we rule out such a model of compactification that there exists a Kaluza-Klein mode satisfying $2m_{KK} = \omega_b$, where m_{KK} is mass of the Kaluza-Klein mode and ω_b is the frequency of oscillation of the radius of the compact manifold on which extra dimensions are compactified. This is a restatement of the criterion proposed previously [S. Mukohyama, Phys. Rev. **D57**, 6191 (1998)]. As an example, we consider a model of compactification by a sphere and investigate Kaluza-Klein modes of $U(1)$ field. In this case the parametric resonance is so mild that the sphere model is not ruled out.

I. INTRODUCTION

Many unified theories, such as superstring theory [1] and M-theory [2], require the dimensionality of spacetime larger than four. In these theories, we have to reduce spacetime dimensions to obtain a four-dimensional theory describing the observed universe. While a new mechanism was recently proposed by Randall and Sundrum [3] and has been attracting much interests, the Kaluza-Klein prescription of compactification [4] still has been conventionally adopted. In particular, combination of these two mechanism seems promising. In the Kaluza-Klein prescription, extra dimensions are compactified on a compact manifold not to be seen in low energy. As we can easily expect, the corresponding four-dimensional effective theory depends strongly on the way of compactification. This may be considered as an advantage: the four-dimensional theory including abundant contents may emerge from a simple higher-dimensional theory as a result of dynamics of the theory. However, at the same time, it may be considered as a disadvantage: we cannot give definite predictions in four-dimension without specifying the way of compactification.

In this respect, Kolb and Slansky [5] did an interesting work. They considered a five-dimensional theory including gravity and a massless scalar field. After compactifying the extra one-dimension by a circle, they investigated cosmological evolution of energy density ρ_{KK} of Kaluza-Klein modes of the scalar field, provided that entropy production is negligible. It was concluded that, if ρ_{KK} is comparable to radiation energy density ρ_{rad} at an early epoch of the universe, the former density exceeds the critical density of the universe soon. Therefore, quanta of Kaluza-Klein modes must not be excited catastrophically in the early universe.

Their conclusion is essentially because of the momentum conservation along the circle. Because of it, a quantum of a Kaluza-Klein mode cannot decay into zero modes without meeting accidentally with a quantum of another Kaluza-Klein mode with the exactly opposite momentum along the circle. Hence, ρ_{KK} evolves simply like $\propto a^{-3}$, and $\rho_{KK}/\rho_{rad} \propto a$, where a is the scale factor of the universe.

Since the momentum conservation along the compact manifold is expected to hold in a large class of models of compactification, it seems that the above result can be generalized. Namely, *quanta of Kaluza-Klein modes must not be excited so catastrophically that energy density of them become comparable to radiation energy density.*

Purpose of this paper is to analyze whether quanta of Kaluza-Klein modes are excited by parametric resonance caused by small oscillation of the radius of compactification. Such a analysis may give a cosmological criterion on ways of compactification. In Sec. II we review and restate clearly the cosmological criterion proposed in ref. [6]. In Sec. III, based on the criterion, we consider a model of compactification by a sphere and investigate Kaluza-Klein modes of a $U(1)$ field. We conclude that excitation of Kaluza-Klein modes is so mild that the sphere model is not

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ruled out. Sec. IV is devoted to a summary of this paper. In Appendix A we derive hamiltonian for a massive vector field (or a Proca field) for the use in Sec. III.

II. COSMOLOGICAL CRITERION ON COMPACTIFICATION

Let us suppose the following situation: (a) we consider a D-dimensional theory which includes gravity and other fields; (b) we adopt the conventional Kaluza-Klein compactification as a way of dimensional reduction: we compactify the extra $(D-4)$-dimensions on a compact manifold; (c) we consider some mechanism (e.g. the Casimir effect) to stabilize the compactification; (d) we also consider perturbations of D-dimensional fields around the background specified by the Kaluza-Klein prescription.

The perturbations in (d) include Kaluza-Klein modes which can be considered as massive fields in four-dimension. Squared mass of the Kaluza-Klein modes are essentially eigenvalues of the Laplacian on the compact manifold. Hence, it is proportional to b^{-2}, where b is the radius of the compact manifold. On the other hand, (c) implies that b has a potential with at least one local minimum, say at $b = b_0$, when it is considered as a four-dimensional field. Evidently, b_0 can be considered as the present value of b. Thus, it is natural to assume that b oscillates around b_0 in the early universe. The frequency of the oscillation is expected to be of the order of b_0^{-1}. Therefore, it is expected that in the early universe the mass of the Kaluza-Klein modes, which is of the order of b_0^{-1}, also oscillates with the frequency of the order of b_0^{-1}. Although there are freedom of conformal transformation and field redefinitions which will be explained in the next section for a particular model, the above expectation still seems to hold for a large class of compactification. Namely, we can expect that *in the early universe the mass of the Kaluza-Klein mode oscillates with the frequency of the same order as the mass itself.* In such a situation, parametric resonance might occur to enhance creation of quanta of the Kaluza-Klein mode.

In order to analyze the parametric resonance phenomenon in the expanding universe, we need hamiltonian of each Kaluza-Klein mode. Considering the FRW spacetime as the four-dimensional spacetime and performing Fourier transformation (or harmonic expansion) w.r.t. the three-dimensional space, hamiltonian for each Fourier component of a Kaluza-Klein mode is expected to have the following form.

$$H = \frac{1}{2}[P^2 + \Omega^2(t)Q^2], \tag{1}$$

where Q is a Fourier component of a Kaluza-Klein mode, P is its conjugate momentum, and

$$\Omega^2(t) = m_{KK}^2[1 - \epsilon \cos(\omega_b t)] + a^{-2}k^2 + O(\epsilon^2). \tag{2}$$

Here, the constants m_{KK}, ω_b and k^2 are the mass of the Kaluza-Klein mode at $b = b_0$, the frequency of the oscillation of b and the squared comoving momentum along the three-dimensional space, respectively. The scale factor of the FRW universe was written as a, and ϵ is a dimensionless quantity proportional to the amplitude of the oscillation of b. Throughout this paper we assume that $\epsilon \ll 1$, which will be derived from another more essential assumption.

From this form of the hamiltonian, the following is easily expected: if $A + B \approx 1$ then the parametric resonance occurs to enhance the process $b \to KK + \overline{KK}$; if $A + B \approx 4$ then the parametric resonance enhances the process $b \times 2 \to KK + \overline{KK}$; $\cdots\cdots$; if $A + B \approx n^2$ then the process $b \times n \to KK + \overline{KK}$ is enhanced, where A and B are defined by

$$A = 4 \cdot \frac{a^{-2}k^2}{\omega_b^2},$$

$$B = 4 \cdot \left(\frac{m_{KK}}{\omega_b}\right)^2, \tag{3}$$

and b, KK and $\overline{KK}$ represent a quanta of the oscillation of the field b, a quanta of a Kaluza-Klein mode and a quanta of another Kaluza-Klein mode with the exactly opposite momentum along the compact manifold, respectively. This observation follows from the fact that $A + B$ is essentially equal to $(2E_{KK}/\omega_b)^2$, where E_{KK} is energy of the Kaluza-Klein mode. Note that A depends on time while B does not. In particular, $\dot{A}/A = -2H$, where $H \equiv \dot{a}/a$ is the Hubble parameter.

The creation rate of quanta of the Kaluza-Klein mode is given by

$$\Gamma \equiv \frac{d}{dt} \sinh^{-1} \sqrt{N_{KK}} \sim \epsilon^n \omega_b \sim \frac{\epsilon^n}{b_0} \quad \text{for } \left|(A+B) - [n^2 + O(\epsilon^2)]\right| < O(\epsilon^n) \ (n = 1, 2), \tag{4}$$

and otherwise $\Gamma \equiv (d/dt) \sinh^{-1} \sqrt{N_{KK}} \sim 0$, where N_{KK} is the number of created quanta [6]. By using this formula, we can estimate N_{KK}. For a mode satisfying $(n-1)^2 < B < n^2$ and $B - n^2 = O(1)$, the duration Δt of the parametric resonance is estimated to be $\sim H^{-1} \epsilon^n$. Thus, integration of (4) gives

$$\sinh^{-1} \sqrt{N_{KK}} \sim \Gamma \cdot \Delta t \sim \frac{\epsilon^{2n}}{Hb_0} \sim \epsilon^{2n-1} \sqrt{\frac{\rho_b}{\rho_0}}, \tag{5}$$

where $\rho_b \sim \epsilon^2 b_0^{-2}/\kappa^2$ and $\rho_0 \equiv 3H^2/\kappa^2$ are energy density of the oscillation of b and the critical density of the universe, respectively. Here, κ^2 is the four-dimensional gravitational coupling constant. To obtain this estimate, we used the adiabatic approximation, assuming that $Hb_0 \ll 1$ and $\dot{H}b_0^2 \ll 1$. Note that the previous assumption $\epsilon \ll 1$ can be derived from the assumption $Hb_0 \ll 1$, since $\rho_b/\rho_0 \sim (\epsilon/Hb_0)^2$. On the other hand, for $B = n^2$, the duration Δt of the parametric resonance is $\sim H^{-1}$, and thus the integration of (4) gives

$$\sinh^{-1} \sqrt{N_{KK}} \sim \Gamma \cdot \Delta t \sim \frac{\epsilon^n}{Hb_0} \sim \epsilon^{n-1} \sqrt{\frac{\rho_b}{\rho_0}}. \tag{6}$$

We used the adiabatic approximation to obtain this result, too. Since $\rho_b \leq \rho_0$, the r.h.s.'s of (5) and (6) are bounded from above by $O(\epsilon^{2n-1})$ and $O(\epsilon^{n-1})$, respectively. Therefore, N_{KK} can be of order unity if and only if $B = 1$.

Hence, let us estimate created energy density for the case $B = 1$. First, by definition of A the typical value of k^2 in the resonance band is $\sim \Delta A \cdot a^2 \omega_b^2$, where ΔA is the band width $\sim \epsilon$. Next, the energy density ρ_{KK} of created quanta of the Kaluza-Klein mode is $\sim m_{KK}(k^2)^{3/2}N_{KK}$. Thus,

$$\frac{\rho_{KK}}{\rho_b} \sim \left(\frac{\kappa}{b_0}\right)^2 \epsilon^{-1/2} N_{KK}. \tag{7}$$

Now let us consider backreaction, which we have not yet taken into account. First, if we take (6) and (7) at their face value then they imply that $\rho_{KK} \gg \rho_b$, provided that $\rho_b \sim \rho_0$ initially and that b_0 is of Planck order $\sim \kappa$. Next, the backreaction becomes important when and only when ρ_{KK} becomes comparable to ρ_b. Hence, this rather overestimated result obtained without considering backreaction meant that $\rho_{KK} \sim \rho_b$ ($\sim \rho_{rad}$) if the backreaction was taken into account. Therefore, from the argument in the previous section, we have to rule out models of compactification in which there exists a Kaluza-Klein mode with $B = 1$, which is equivalent to $2m_{KK} = \omega_b$.

In the above argument we have ignored interaction among Kaluza-Klein modes. This treatment can be justified, provided that coupling constants are at most of order unity in the unit of b_0. Actually, in this case we can expect that interaction should be small unless $\rho_{KK} \sim b_0^{-4}$ ($\gg \rho_b$). Therefore, the result obtained above without considering interaction is not altered by inclusion of interaction.

Finally, we propose the following cosmological criterion on models of compactification: *we rule out such a model of compactification that there exists a Kaluza-Klein mode satisfying*

$$2m_{KK} = \omega_b, \tag{8}$$

where m_{KK} is mass of the Kaluza-Klein mode and ω_b is the frequency of oscillation of the radius of the compact manifold on which the extra dimensions are compactified.

III. ANALYSIS OF $U(1)$ FIELD

In this section we analyze Kaluza-Klein modes of $U(1)$ field $\bar{A}_M$ described by the action

$$I_{U(1)} = -\frac{1}{4} \int d^D x \sqrt{-\bar{g}} \bar{F}_{MN} \bar{F}^{MN}, \tag{9}$$

where $\bar{F}_{MN} = \partial_M \bar{A}_N - \partial_N \bar{A}_M$. As the compact manifold on which the extra $(D-4)$-dimensions are compactified, we adopt the d-dimensional sphere ($d = D - 4$):

$$\bar{g}_{MN} dx^M dx^N = \left(\frac{b}{b_0}\right)^{-d} g_{\mu\nu} dx^\mu dx^\nu + b^2 \Omega_{ij} dx^i dx^j, \tag{10}$$

where the four-dimensional metric $g_{\mu\nu}$ and the radius b depend only on the four dimensional coordinates $\{x^\mu\}$, and $\Omega_{ij} dx^i dx^j$ is the line element of the unit d-sphere.

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To analyze the $U(1)$ field in the background spacetime, it is convenient to expand the field in terms of harmonics on the $(D-4)$-dimensional sphere.

$$\bar{A}_M dx^M = b_0^{-d/2} \left[U_\mu Y dx^\mu + b_0(\phi_T V_{(T)i} + \phi_L V_{(L)i}) dx^i \right], \tag{11}$$

where U_μ, ϕ_T and ϕ_L depend only on the four-dimensional coordinates $\{x^\mu\}$; Y and $V_{(T)i}$ are the scalar harmonics and the transverse-traceless vector harmonics, and $V_{(L)i} = \partial_i Y$. (See Appendix of ref. [8] for definitions and properties of these harmonics.) Hereafter, we omit the summations w.r.t. eigenvalues and in eigenstates. By substituting the expansion into the action, we obtain the corresponding decomposition of the action $I_{U(1)} = I_T + I_U$, where

$$I_T = -\frac{1}{2} \int dx^4 \sqrt{-g} \left[\left(\frac{b}{b_0}\right)^{-2} g^{\mu\nu} \partial_\mu \phi_T \partial_\nu \phi_T + \left(\frac{b}{b_0}\right)^{-(d+4)} \frac{l(l+d-1)+d-2}{b_0^2} \phi_T^2 \right],$$

$$I_U = -\int dx^4 \sqrt{-g} \left[\frac{1}{4} \left(\frac{b}{b_0}\right)^{d} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} + \frac{1}{2} \left(\frac{b}{b_0}\right)^{-2} \frac{l(l+d-1)}{b_0^2} g^{\mu\nu} U_\mu U_\nu \right], \tag{12}$$

and $F_{\mu\nu} = \partial_\mu U_\nu - \partial_\nu U_\mu$. Note that ϕ_L represents gauge degrees of freedom and that $I_{U(1)}$ does not depend on it.

From (12), we can see that masses m_T and m_U for the four-dimensional fields ϕ_T and U_μ are given by

$$m_T^2 = \frac{l(l+d-1)+d-2}{b_0^2},$$

$$m_U^2 = \frac{l(l+d-1)}{b_0^2}. \tag{13}$$

(See Appendix A for a systematic treatment of the field U_μ in the FRW universe.) On the other hand, if we assume that the compactification is stabilized by the Casimir effect [7], then d should be larger than 1 and the frequency ω_b of the oscillation of b is equal to 2ω in ref. [6,8]. Hence, $\omega_b^2 = 2(d-1)/b_0^2$, and

$$\left(\frac{2m_T}{\omega_b}\right)^2 = \frac{2[l(l+d-1)+d-2]}{d-1} > 1 \quad (l \geq 1),$$

$$\left(\frac{2m_U}{\omega_b}\right)^2 = \frac{2l(l+d-1)}{d-1} > 1 \quad (l \geq 1). \tag{14}$$

Finally, from the argument in the previous section, we conclude that the parametric resonance of Kaluza-Klein modes of the $U(1)$ field is not so catastrophic. Hence, the analysis of this section does not rule out the model of compactification by the d-dimensional sphere with the Casimir effect.

IV. SUMMARY AND DISCUSSIONS

In summary, we have analyzed parametric resonance of Kaluza-Klein modes and investigated whether a model of compactification can survive or not. First, based on the theory of parametric resonance, we have restated the cosmological criterion proposed in ref. [6] in the following form: *we rule out such a model of compactification that there exists a Kaluza-Klein mode satisfying*

$$2m_{KK} = \omega_b, \tag{15}$$

where m_{KK} is mass of the Kaluza-Klein mode and ω_b is the frequency of oscillation of the radius of the compact manifold on which the extra dimensions are compactified. Next, as an example, we have considered a model of compactification by a sphere and investigated Kaluza-Klein modes of $U(1)$ field. We have concluded that parametric resonance is so mild that the sphere model is not ruled out.

In order to obtain the above criterion, we have assumed that $Hb_0 \ll 1$ and $\dot{H}b_0 \ll 1$. In future works, these assumptions should be removed, and then we should inevitably consider the broad resonance regime $\epsilon \sim 1$ [1]. In this

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[1] In this respect, ref. [10] may be considered as the first step.

regime, the so called stochastic resonance occurs [9], and the structure of resonance depends not only on the curvature of the potential of the field b at a local minimum (or ω_b) but also on the whole shape of the potential.

Although we have concluded that Kaluza-Klein modes of the $U(1)$ field are not excited catastrophically by parametric resonance and that the sphere model of compactification is not ruled out, still there may be possibilities that Kaluza-Klein modes of other fields might be excited strongly by parametric resonance. In this respect, in ref. [6,8], Kaluza-Klein modes of a massless scalar field and a part of Kaluza-Klein modes of gravitational perturbations were analyzed. It was concluded that parametric resonance of those Kaluza-Klein modes is also mild.

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APPENDIX A: PROCA FIELD IN FRW UNIVERSE

In this appendix we consider a massive vector field (or a Proca field) U_μ in the $(n+1)$-dimensional FRW universe. The action is

$$I = -\int dx^{n+1}\sqrt{-g}\left[\frac{f}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} + \frac{m^2}{2}g^{\mu\nu}U_\mu U_\nu\right], \tag{A1}$$

where $F_{\mu\nu} = \partial_\mu U_\nu - \partial_\nu U_\mu$, and the background is given by

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\Sigma_{pq}dx^p dx^q,$$
$$f = f(t),$$
$$m^2 = m^2(t). \tag{A2}$$

Here, $\Sigma_{pq}dx^p dx^q$ is the line element of a n-dimensional space of constant curvature, whose curvature constant is denoted by K in the following argument. By substituting these background into the action, we obtain

$$I = \frac{1}{2}\int dt dx^n \sqrt{\Sigma}\left[af\Sigma^{pq}(\dot{A}_p - \partial_p A_0)(\dot{A}_q - \partial_q A_0) + a^3 m^2 A_0^2\right.$$
$$\left. - \frac{f}{a}(\Sigma^{pp'}\Sigma^{qq'} - \Sigma^{pq'}\Sigma^{qp'})\partial_p A_q \partial_{p'} A_{q'} - am^2\Sigma^{pq}A_p A_q\right], \tag{A3}$$

where the dot denotes a time derivative. Since the action does not include $\dot{A}_0$, $\delta I/\delta A_0 = 0$ gives a constraint. Actually, the constraint is the second class and can be solved formally as

$$U_0 = (\Delta - f^{-1}a^2 m^2)^{-1}\Sigma^{pq}\partial_p \dot{U}_q, \tag{A4}$$

where Δ is the Laplacian in the n-dimensional space of constant curvature. This can be used to eliminate A_0 from the action.

In order to obtain explicit expression for the r.h.s. of (A4), we expand the field U_μ in terms of harmonics on the n-dimensional space of constant curvature.

$$U_\mu dx^\mu = u_0 y dt + (u_\perp v_{(T)p} + u_\parallel v_{(L)p})dx^p, \tag{A5}$$

where u_0, $u_\perp$ and $u_\parallel$ depend only on the time variable t; y and $v_{(T)p}$ denote the scalar harmonics and the transverse-traceless vector harmonics, respectively, and $v_{(L)p} = \partial_p y$. Hereafter we omit the summations (or integrations) w.r.t. eigenvalues and in eigenspaces. (For definition of harmonics, we adopt those given in Appendix of ref. [11].) Correspondingly, the action becomes the following form.

$$I = \frac{1}{2}\int dt\left\{af[\dot{u}_\perp^2 + k^2(\dot{u}_\parallel - u_0)^2] + a^3 m^2 u_0^2 - a^{-1}f[k^2 + (n-1)K]u_\perp^2 - am^2(u_\perp^2 + k^2 u_\parallel^2)\right\} \tag{A6}$$

The constraint (A4) can be rewritten as

$$u_0 = \frac{fk^2 \dot{u}_\parallel}{fk^2 + a^2 m^2}. \tag{A7}$$

By using this expression, we can eliminate u_0 in the action to give

$$I = \int dt (L_\perp + L_\parallel),$$
$$L_{\perp,\parallel} = \frac{1}{2} \left(\dot{Q}_{\perp,\parallel}^2 - \Omega_{\perp,\parallel}^2 Q_{\perp,\parallel}^2 \right), \tag{A8}$$

where $Q_\perp$ and $Q_\parallel$ are defined by

$$Q_\perp = u_\perp \sqrt{af},$$
$$Q_\parallel = u_\parallel \sqrt{\frac{afk^2}{1 + fm^{-2}a^{-2}k^2}}, \tag{A9}$$

and

$$\Omega_\perp^2 = f^{-1}m^2 + [k^2 + (n-1)K]a^{-2} - \Delta_\perp^2/2 - \dot{\Delta}_\perp,$$
$$\Omega_\parallel^2 = f^{-1}m^2 + k^2 a^{-2} - \Delta_\parallel^2/2 - \dot{\Delta}_\parallel. \tag{A10}$$

Here, $\Delta_\perp$ and $\Delta_\parallel$ are defined by

$$\Delta_\perp = \frac{d}{dt} \ln(af),$$
$$\Delta_\parallel = \frac{d}{dt} \ln \left(\frac{af}{1 + fm^{-2}a^{-2}k^2} \right). \tag{A11}$$

Therefore, hamiltonians for $Q_\perp$ and $Q_\parallel$ w.r.t. time t are

$$H_{\perp,\parallel} = \frac{1}{2} \left(P_{\perp,\parallel}^2 + \Omega_{\perp,\parallel}^2 Q_{\perp,\parallel}^2 \right), \tag{A12}$$

where $P_{\perp,\parallel}$ are momenta conjugate to $Q_{\perp,\parallel}$, respectively.

Setting $n = 3$, $f = (b/b_0)^d$ and $m^2 = (b/b_0)^{-2}m_U^2$, we can apply the above result to the Kaluza-Klein mode in Sec. III. Actually, $\Omega_{\perp,\parallel}^2$ becomes of the form.

$$\Omega_{\perp,\parallel}^2 = m_U^2 [1 - \epsilon \cos(\omega_b t)] + a^{-2} \tilde{k}^2 + O(\epsilon^2), \tag{A13}$$

where $\tilde{k}^2$ is $k^2 + 2K$ for $\Omega_\perp$ and k^2 for $\Omega_\parallel$, respectively, and ϵ is a dimensionless quantity proportional to the amplitude of oscillation of b around b_0. To derive this expression we have assumed that $Hb_0 \ll 1$ and $\dot{H}b_0^2 \ll 1$.

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